The causal structure could be different from the best posterior probability structure

Prof Jan Kruger

Unisa graduate school for business leadership - MIDRAND

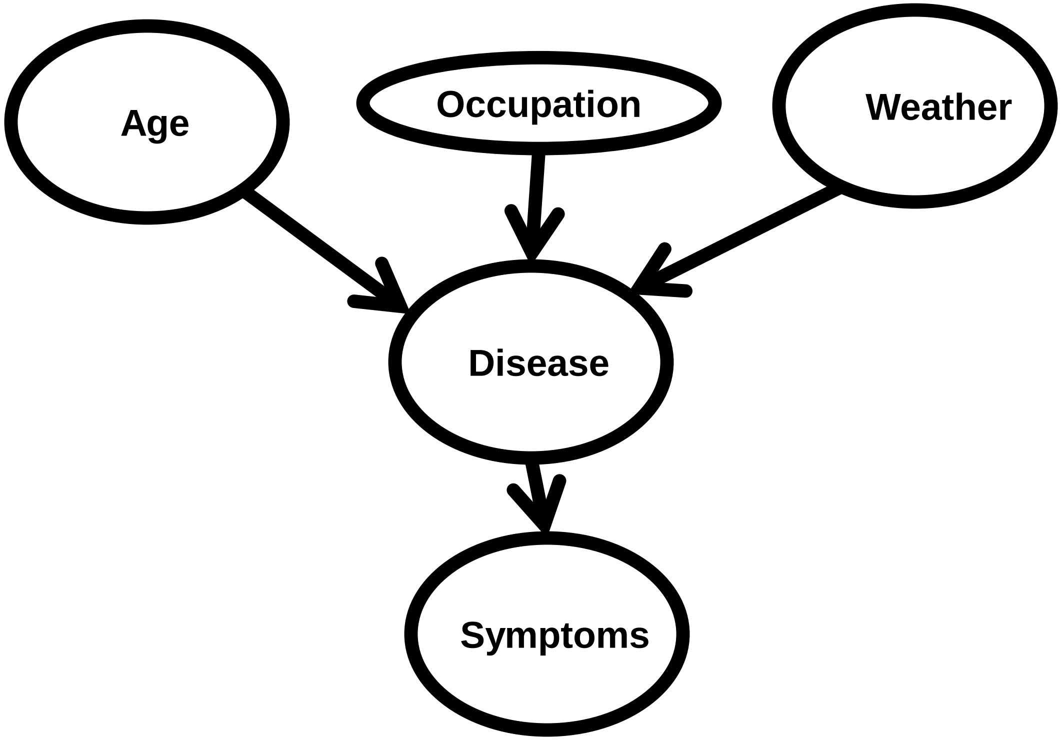
Business school/ finance case

- Keynote address presented at Finance conferences
- Causality used in Investment selection
- Causality can explain Financial distress
- Causality can be used to forecast VaR
- Causality is a tool to explain phenomena

Prof Jan Kruger
Krugejw@unisa.ac.za

Simplified Bayesian Structure

P(Age,Occupation,Weather,Disease,Symptoms)=P(Symptoms|Disease)P(Disease|Age,Occupation,Weather)P(Age)P(Occupation)P(Symptoms)
(Buntine, 1996)



Prof Jan Kruger
Krugejw@unisa.ac.za

Cooper and Herskovits, 1989

$$p(D \mid S^h) = \prod_{i=1}^{n} \prod_{j=1}^{q} \frac{\Gamma(\alpha_{ij})}{\Gamma(N_{ij} + \alpha_{ij})} \prod_{k=1}^{r} \frac{\Gamma(N_{ijk} + \alpha_{ijk})}{\Gamma(\alpha_{ijk})}$$

Prof Jan Kruger
Krugejw@unisa.ac.za

Number of structures

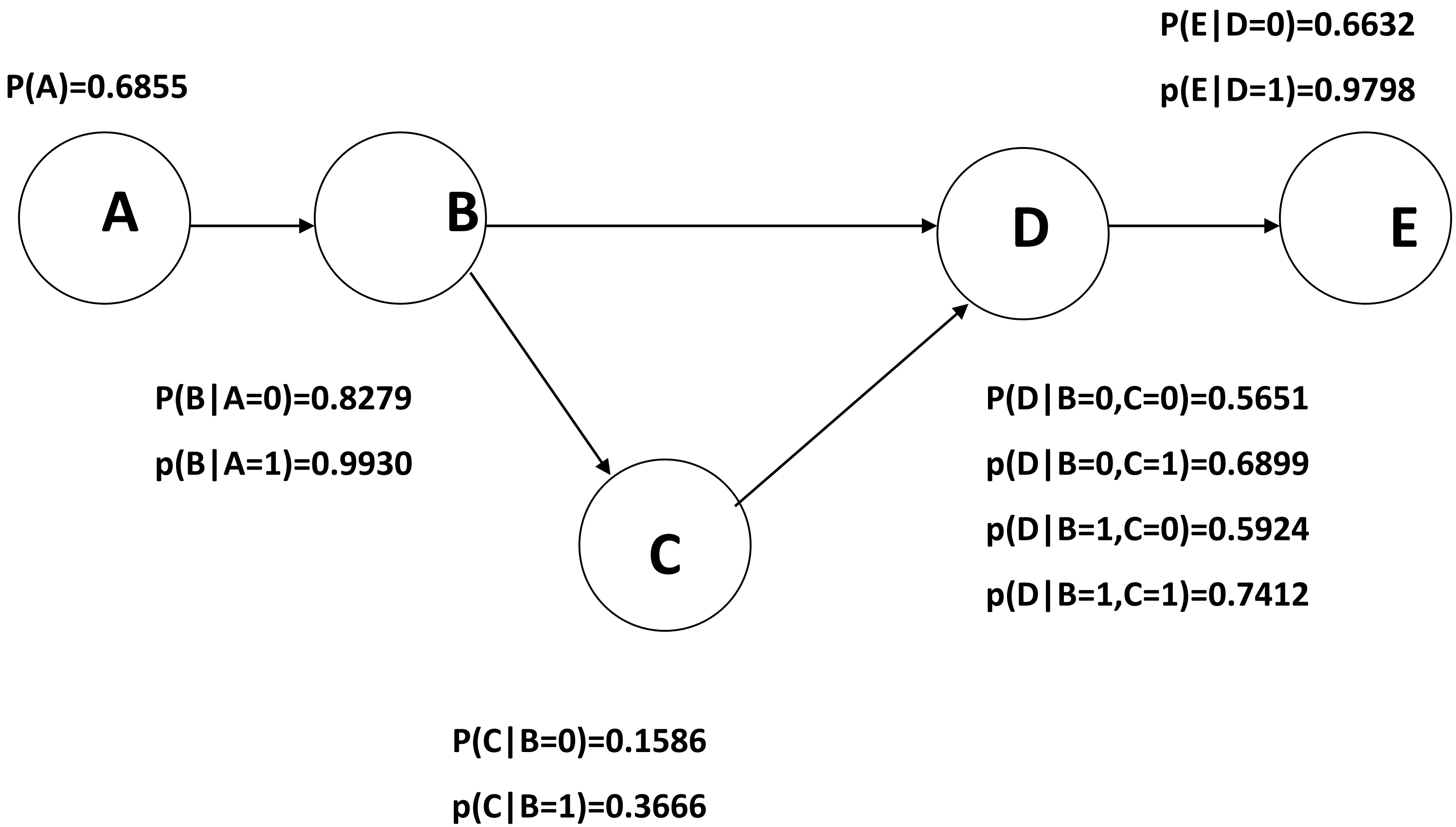
$$3^{\frac{n(n-1)}{2}}$$

Prof Jan Kruger
Krugejw@unisa.ac.za

Steps

- Select a model
- Create a frequency distribution
- Do an exhaustive search over structures
- To find the max posterior probability structure
- It must be the same as the structure that created the data.

Prof Jan Kruger
Krugejw@unisa.ac.za



Prof Jan Kruger
Krugejw@unisa.ac.za

Best posterior model is not the causal model

- The unscaled log posterior probability for the correct model is -23755. The models with the best unscaled log posterior probability of -23750 are A→B→C→D→E, A←B→C→D→E, A←B←C→D→E, A←B←C←D→E and A←B←C←D←E. This gives a posterior probability that is e^5 times larger for the wrong model.

New norm

Path between nodes A-B-C

Norm $\dfrac{\rho_{AB}\rho_{BC}}{\rho_{AC}}$

Prof Jan Kruger
Krugejw@unisa.ac.za

Given variables A, B and C and assume only linear relationships, then the norm of the path(A-B-C) = 1 if and only if A and C are conditionally independent given B or I(A, C| B)

If the norm of the path A-B-C equals one, then

$$\frac{\rho_{AB}\rho_{BC}}{\rho_{AC}} = 1 \quad \text{or} \quad \rho_{AB}\rho_{BC} = \rho_{AC}$$

Only linear relationships are considered,

$$\rho_{AC.B} = \frac{\rho_{AC} - \rho_{AB}\rho_{BC}}{\sqrt{(1-\rho_{AB}^2)(1-\rho_{BC}^2)}} = 0$$

therefore A and C are conditionally independent given B or I(A, C | B).

The converse: If A and C are conditionally independent given B, I(A, C | B) then $\rho_{AB}\rho_{BC} = \rho_{AC}$.

Elimination heuristic

- Start with a complete graph containing all the variables.
- Eliminate arcs between variables that are independent (with a correlation coefficient not significantly different from zero).
- Eliminate the arcs where Conditional Independence Statements have been identified (where the norm equals one).
- This gives us the causal tree structure, and I tested that it is also the structure with the highest posterior probability.

Prof Jan Kruger
Krugejw@unisa.ac.za

Theorem 1

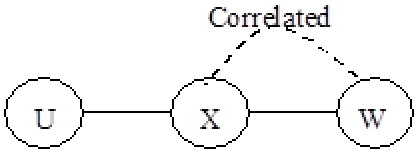
- If two variables, X and W, are correlated and the path U-X-W is a valid path, then the path with X and W interchanged, U-W-X will have a norm equal to the coefficient of determination (the square of the correlation coefficient) between X and W

Prof Jan Kruger
Krugejw@unisa.ac.za

If u-x-w valid path then norm of u-w-x = ρ^2

-

Figure 4.5 Correlated variables on a valid path



Norm(U-X-W) = 1

Norm(U-W-X)= Coeff. Det. Between X and W

Prof Jan Kruger
Krugejw@unisa.ac.za

If u-x-w valid path then norm of u-w-x = ρ^2

- **Proof**: A valid (star decomposable) path U-X-W means that
- ,or, $\rho_{UW} = \rho_{UX} \cdot \rho_{XW}$
- Therefore the norm for the path U-W-X is
- This proves the theorem.
- The following theorem can be used to check if we have found a valid path.

Theorem 2

- X-Y-Z is a valid and only path between X and Z if and only if the product of the norms is equal to the coefficient of determination between the start and end nodes of this path.
- Norm(X-Y-Z).Norm(Y-X-Z).Norm(X-Z-Y) $= \rho^2{}_{XZ}$.

SBL GRADUATE SCHOOL OF BUSINESS LEADERSHIP UNISA

Prof Jan Kruger
Krugejw@unisa.ac.za

Proof

- Assume a valid and only path X-Y-Z. Then, for a valid and only path there is a tree structure between the variables and the norm will be one (in other words $\rho_{XY}\rho_{XZ} = \rho_{YZ}$).
- Therefore $$\frac{\rho_{XY}\rho_{YZ}}{\rho_{XZ}}\frac{\rho_{XY}\rho_{YZ}}{\rho_{YZ}}\frac{\rho_{XZ}\rho_{YZ}}{\rho_{XY}} = \rho_{XY}\rho_{XZ}\rho_{YZ} = \rho_{YZ}^2$$

  - In the opposite direction: if the product of the norms is equal to the coefficient of determination, then for the path A-E-C this condition can be written as: $$\frac{\rho_{AE}\rho_{EC}}{\rho_{AC}}\frac{\rho_{AC}\rho_{CE}}{\rho_{AE}}\frac{\rho_{AE}\rho_{AC}}{\rho_{EC}} = \rho_{AC}^2$$
- If the correlation coefficient between A and C is not zero, then
- $\therefore \rho_{EC}\rho_{AE}\rho_{AC} = \rho_{AC}^2$ or A-E-C is a valid and only path.